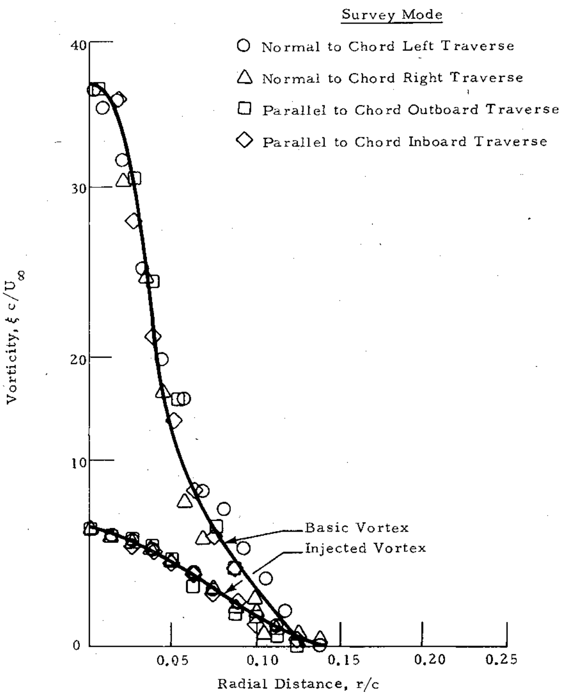

Fig. 2 Vorticity of trailing vortex measured by vorticity meter.

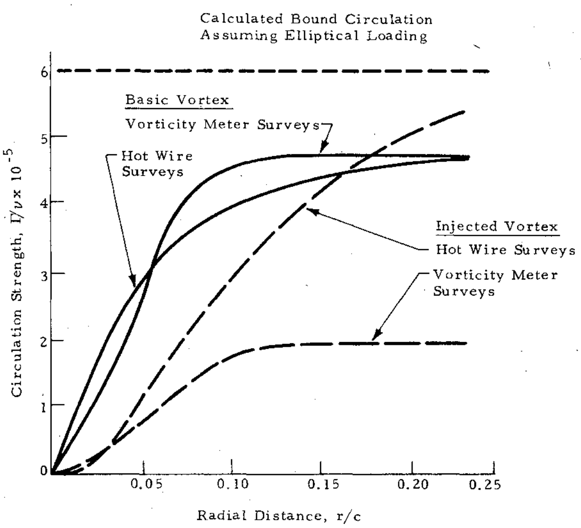

Fig. 3 Comparison of vortex circulation strength calculated from vorticity meter and hot-wire measurements.

the vortex as shown by the hotwire, laser velocimeter, and pressure probe measurements. On the other hand, the vorticity meter surveys indicate the opposite trend, a decrease in the core circulation with vortex injection. Thus, the vorticity meter measurements are not consistent with the hot-wire surveys or with the trends obtained by other measurement techniques.

Although vorticity meters have been used often to measure vortex flows, no study has been done to compare the vorticity meter with other flow measuring devices. Comparisons of the vorticity meter and hot-wire system discussed in this note indicate that the vorticity meter behaves in a nonlinear fashion for weak vortex flows. This may explain the low vortex circulations cited in Refs. 4-6 and avert future misunderstandings.

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Prediction of Turbulent Boundary Layers at Low Reynolds Numbers

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Introduction

FINITE-DIFFERENCE calculation methods, using eddy-viscosity or mixing length models, have become reasonably well established as efficient and reliable tools for predicting most features ot two-dimensional turbulent wall boundary layers under a fairly wide range of conditions. However, some uncertainty apparently still exists regarding the nature of turbulent flow at low Reynolds numbers, and the form of the turbulence model required to accurately predict this flow using a finite-difference calculation procedure.

Although numerous turbulent models which have been proposed differ in detail, many share the use of damped mixing length evaluation for the turbulent viscosity in the inner region according to the form

$$\mu_{T(\text{inner})} = \rho \ell^2 \, |\partial u / \partial y| \tag{1}$$

with ℓ being specified for at least part of the inner region as

$$\ell_{(\text{inner})} = \kappa D y \tag{2}$$

where κ is the von Karman constant and D is a damping function which accounts for the effects of the kinematic viscosity on the turbulence near the wall. To a large extent in the outer region, the majority of the simple models employ either a

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Clauser type eddy viscosity formulation[1]

$$\mu_{T(\text{outer})} = \alpha \rho u_e \delta_k^* \tag{3}$$

where δ_k^* is the kinematic displacement thickness and the α function includes any intermittancy effects or Reynolds number dependence, or a mixing length formulation, indicated by Eq. (1), with ℓ given as

$$\ell_{(\text{outer})} = C\delta \tag{4}$$

for at least the outer-most portion of the boundary layer. In this Note, we will refer to those models which employ the Clauser type formulation [Eq. (3)] as Type 2 models. Examples of predictions made by Type 1 models can be found in Refs. 1 and 2, and examples of predictions by Type 2 models in Refs. 3-5.

Phenomenological models of this simplicity cannot be expected to accurately predict all details of the turbulent motion; however, they have proven adequate for a variety of flows important in engineering applications and warrant further evaluation and study on that basis. Several investigators have recently reported the need to make special modifications to their turbulence models in order to obtain accurate predictions at low Reynolds numbers. In at least three instances,[2,6,7] an empirical function of Reynolds number was incorporated into turbulence models to account for low Reynolds number phenomena. Cebeci[7] made a particularly convincing demonstration that the α in Type 1 models should be expressed as a function of Re_θ. Although there have been proposals to the contrary[8], Cebeci's results[7] and the study by Huffman and Bradshaw[9] tend to substantiate that κ remains at a constant value near 0.41 even for turbulent flows at low Reynolds numbers.

The purpose of the present Note is to shed further light on the uncertainties surrounding the prediction of boundary layers at low Reynolds number and to demonstrate some of the attractive features of Type 2 models which permit the accurate prediction of low Reynolds number flows without recourse to model alterations in the form of Reynolds number functions.

Analysis

Good results have been observed to date[5] using the Type 2 model in its simplest form (some previous calculations have used $\kappa = 0.42$) with the mixing length distribution given by:

Model 2A

$$\ell/\delta = 0.41(1-e^{-y^+/26})y/\delta, \qquad y/\delta \le 0.089/0.41(1-e^{-y^+/26}) \tag{5}$$

$$\ell/\delta = 0.089, \qquad y/\delta > 0.089/0.41(1-e^{-y^+/26}) \tag{6}$$

Qualitatively it is correct to think of the inner model [Eq.(5)] as causing the logarithmic, buffer, and viscous sublayer portions of the velocity profile and the outer model [Eq.(6)] as causing the wake-like portion of the velocity profile.

A generalization of the model suitable for flows with transpiration and pressure gradients was given in Ref. 10. In most instances the pressure gradient effect is small, and Eqs. (5) and (6) appear to do fairly well in moderate pressure gradients.

In Fig.1, the skin friction coefficient predicted by Model 2A using the finite-difference method described in Refs. 4 and 5 is compared with experimental data for incompressible, two-dimensional turbulent flow with zero pressure gradient. The agreement between predictions and measurements is reasonably good even at low Reynolds numbers. No low Reynolds number correction appears necessary. The low Reynolds number data of Simpson[11] were omitted from this comparison since more recent investigators[12,13] suggested that the values of c_f presented by Simpson were in error.

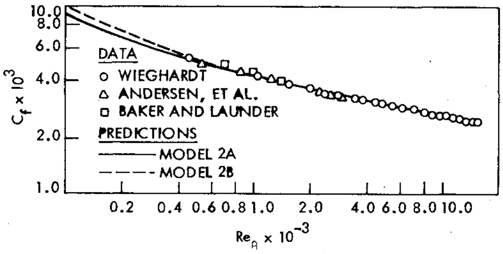

Fig.1 Comparison of predicted and experimental[13,15,16] skin-friction coefficients for an incompressible turbulent boundary layer on a flat plate at low Reynolds numbers.

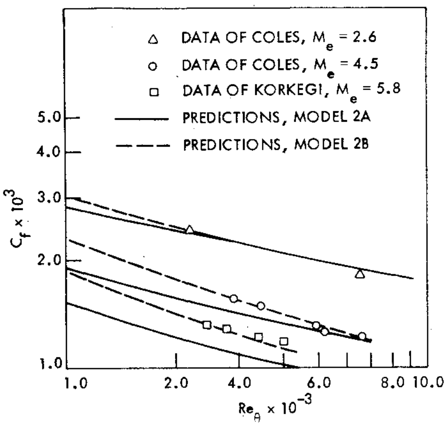

Fig. 2 Comparison of predicted skin-friction coefficients with the measurements of Coles[14] and Korkegi[17] for the compressible turbulent boundary layer on a flat plate at low Reynolds numbers.

Despite this good level of agreement, the implications of the model at very low Reynolds numbers are bothersome. Since these flows are characterized by relatively thinner boundary layers, the value of y^+ at which the switch from the inner [Eq. (5)] to the outer [Eq. (6)] model (and the y^+ at which the wake component of the profile will begin to develop) will be smaller and smaller as decreasing values of Re_θ are considered. At sufficiently low values of Re_θ, the switch-over point will occur while the mixing length is still being damped (say for $y^+ < 50$), so that the characteristic logarithmic (fully turbulent) portion of the velocity profile will hardly begin to develop before the wake-like mixing length distribution of Eq. (6) becomes effective. According to the present study, this would only occur for Re_θ less than about 400 for incompressible flow, but the tendency for the fully turbulent region of the flow to be truncated is found to persist to higher and higher values of Re_θ in compressible flow in response to effects of variable properties as the Mach number increases. This suppression of the fully turbulent region is counter to the preponderance of measurements in turbulent flow which indicates the existence of a region for which $\ell = \kappa y$.

A slight modification to Type 2 models is now proposed to eliminate this anomalous behavior at low Reynolds numbers. With no experiemental evidence known to exist to the contrary, it is assumed (1) that the form of the damping function and the value of the von Karman constant are independent of Reynolds number and (2) that the mixing length will closely approach the fully turbulent value prior to becoming independent of distance from the wall, that is, no wake-like region will form until a fully turbulent region exists. These hypotheses can be used in the following manner:

Model 2B

To ensure that fully turbulent-like effective viscosities exist in the flow, the switch-over from this inner model [Eq. (5)] to the outer model [Eq. (6)] will not be permitted until $y^+ = 50$, at which point the damping function has reached 0.85 in value, even if the inner model predicts $\ell/\delta > 0.089$. If at $y^+ = 50$ the inner model [Eq. (5)] predicts $\ell/\delta \leq 0.089$, then all aspects of the model are exactly as given by Model 2A. If, however, at $y^+ = 50$, Eq (5) predicts $\ell/\delta < 0.089$, then the mixing length in the outer region becomes constant at

$$\ell = 0.41(1-e^{-50/26})50\mu_w/\rho_w(\tau_w/\rho_w)^{1/2} \tag{7}$$

which is the value of ℓ at $y^+ = 50$ according to Eq. (5). The choice of $y^+ = 50$ in Eq. (7) was arbitrary, but the motivating concept of preserving the fully turbulent (logarithmic) portion of the velocity profile is supported by available low Reynolds number velocity profile measurements.[14-17]

The results of using Model 2B for incompressible turbulent flow are shown in Fig. 1. Model 2B predicts skin friction coefficients which are slightly higher (6% for $Re_\theta = 450$) for $Re_\theta <$ 850 than those predicted by Model 2A; however, from the available data a conclusive case cannot be made for the accuracy of either model over the other. For compressible flow, however, the predictions of Model 2B are dramatically superior to Model 2A at low Reynolds numbers. A representative comparison with the skin-friction coefficient measurements of coles[14] for nominal Mach numbers of 4.5 and 2.6 and Korkegi[17] for a Mach number of 5.8 is shown in Fig. 2. The point of divergence between the two models is seen to translate to larger values of Re_θ as the Mach number increases.

Velocity profiles are predicted quite well at low Reynolds numbers by Type 2 models although space limitations do not permit examples to be presented here. The proposed low Reynolds number modification makes no discernable difference to the predicted velocity profiles for incompressible flows but Model 2B noticeably outperforms model 2A for low Reynolds number compressible flows.

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Concentrated Vortex on the Nose of an Inclined Body of Revolution

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Introduction

WERLE[1] published in 1962 a series of experimental, incompressible surface-flow patterns over both pointed and blunted bodies of revolution at various incidences. In one of these pictures, there appears a vortex pattern on the leeside front nose (Fig. 1a). The accompanying sketch (Fig. 1b) made by Werle shows the limiting streamlines and a clockwise vortex.

Werle did not discuss how, and under what conditions, such a vortex would occur. Since then, little additional understanding of this problem has been developed. Furthermore, it appears that no similar vortex has been reported in later experiments of surface-flow visualization on similar configurations, for example, those by Stetson[2] and by Zakkay et al.[3], among many others. The latter fact has led to suspicion as to whether the vortex reported by Werle is real, or is caused merely by unsteadiness of the test condition.

Recently one of us (Hsieh)[4] carried out experiments of surface-flow visualization for a hemisphere-cylinder by injecting oil through the model surface at Mach numbers 0.6-1.4 and at incidence angles of 0-19°. A vortex pattern similar to that reported by Werle was observed at higher Mach numbers (1-1.4) and higher incidences (15-19°). Samples are shown in Figs. 2a and 2b.

Proposed Mechanism

In this Note, we propose a possible mechanism for the formation of such a nose vortex within the context of the open-and-closed separation idea. Several years ago, one of us (Wang)[5] pointed out that the separation pattern over an elongated body of revolution (for example, an ellipsoid of revolution) differs from prior conceptions (Fig. 3a). It actually changes from a closed separation at low incidence (Fig.

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